Stochastic Quantisation of Gauge Theories

B. Bringmann, S. Cao, M. Hairer, W. Zhao

EPFL / IMPERIAL

Probabilistic paths to QFT annual meeting 2026

Euclidean QFT

Basic construction: Consider a functional S (action) on a space of fields. (Bosonic) Euclidean QFT boils down to constructing the measure

$$\mu_\beta(D\varphi) = e^{-\beta S(\varphi)}\, D\varphi\ .$$

Above expression completely formal since Lebesgue measure $D\varphi$ on space of fields makes no sense. Hope that it yields a well-defined probability measure by some approximation procedure if S is coercive enough.

Osterwalder–Schrader ('75): For local S, construction of "nice enough" μ_β yields genuine QFT with Hilbert space, creation / annihilation operators, time evolution, etc.

Euclidean QFT

Basic construction: Consider a functional S (action) on a space of fields. (Bosonic) Euclidean QFT boils down to constructing the measure

$$\mu_\beta(D\varphi) = e^{-\beta S(\varphi)}\, D\varphi\;.$$

Above expression completely formal since Lebesgue measure $D\varphi$ on space of fields makes no sense. Hope that it yields a well-defined probability measure by some approximation procedure if S is coercive enough.

Osterwalder–Schrader ('75): For local S, construction of “nice enough” μ_β yields genuine QFT with Hilbert space, creation / annihilation operators, time evolution, etc.

Euclidean QFT

Basic construction: Consider a functional S (action) on a space of fields. (Bosonic) Euclidean QFT boils down to constructing the measure

$$\mu_\beta(D\varphi) = e^{-\beta S(\varphi)}\, D\varphi\ .$$

Above expression completely formal since Lebesgue measure $D\varphi$ on space of fields makes no sense. Hope that it yields a well-defined probability measure by some approximation procedure if S is coercive enough.

Osterwalder–Schrader ('75): For local S, construction of "nice enough" μ_β yields genuine QFT with Hilbert space, creation / annihilation operators, time evolution, etc.

Stochastic Quantisation

Proposed by Parisi & Wu '81, earliest rigorous works by Jona-Lasinio & Mitter '85.

Basic idea: Consider discrete approximation to Gibbs measure $e^{-\beta S(\varphi)}\, D\varphi$. This is invariant for stochastic evolution

$$d\varphi = -\nabla S(\varphi)\, dt + \sqrt{2/\beta}\, dW ,$$

for W a Brownian motion with covariance structure adapted to the metric determining the gradient ∇.

Hope: Maybe one can pass to the limit for the dynamic? Define EQFT as invariant measure of limiting process. (Very successful for Φ^4_3 and related models.)

Stochastic Quantisation

Proposed by Parisi & Wu '81, earliest rigorous works by Jona-Lasinio & Mitter '85.

Basic idea: Consider discrete approximation to Gibbs measure $e^{-\beta S(\varphi)}\,D\varphi$. This is invariant for stochastic evolution

$$d\varphi = -\nabla S(\varphi)\,dt + \sqrt{2/\beta}\,dW\ ,$$

for W a Brownian motion with covariance structure adapted to the metric determining the gradient ∇.

Hope: Maybe one can pass to the limit for the dynamic? Define EQFT as invariant measure of limiting process. (Very successful for Φ^4_3 and related models.)

Stochastic Quantisation

Proposed by Parisi & Wu '81, earliest rigorous works by Jona-Lasinio & Mitter '85.

Basic idea: Consider discrete approximation to Gibbs measure $e^{-\beta S(\varphi)}\, D\varphi$. This is invariant for stochastic evolution

$$d\varphi = -\nabla S(\varphi)\, dt + \sqrt{2/\beta}\, dW \ ,$$

for W a Brownian motion with covariance structure adapted to the metric determining the gradient ∇.

Hope: Maybe one can pass to the limit for the dynamic? Define EQFT as invariant measure of limiting process. (Very successful for Φ^4_3 and related models.)

Stochastic Quantisation

Proposed by Parisi & Wu '81, earliest rigorous works by Jona-Lasinio & Mitter '85.

Basic idea: Consider discrete approximation to Gibbs measure $e^{-\beta S(\varphi)}\, D\varphi$. This is invariant for stochastic evolution

$$d\varphi = -\nabla S(\varphi)\, dt + \sqrt{2/\beta}\, dW\ ,$$

for W a Brownian motion with covariance structure adapted to the metric determining the gradient ∇.

Hope: Maybe one can pass to the limit for the dynamic? Define EQFT as invariant measure of limiting process. (Very successful for Φ^4_3 and related models.)

Yang–Mills–Higgs Theory

Ingredients of the theory:

- Compact Lie group G with Lie algebra $\mathfrak{g}$ and Ad-invariant scalar product.
- Finite-dimensional Hilbert space $\mathbf{V}$ with orthogonal action of G.
- Space-time $\mathbf{T}^2$ and "mass" $m \in \mathbf{R}$.

Fields of the theory:

- Gauge field: $\mathfrak{g}$-valued one-forms A on $\mathbf{T}^2$ (actually G-equivariant connections on trivial G-bundle $\mathbf{T}^2 \times G$).
- Higgs field: $\mathbf{V}$-valued field Φ on $\mathbf{T}^2$.

Gauge group $\mathcal{G}$:

- Functions $g\colon \mathbf{T}^2 \to G$ with pointwise multiplication.
- Action on fields: $g \bullet A = \mathrm{Ad}_g A - (dg)g^{-1}$, $g \bullet \Phi = g\Phi$.

Yang–Mills–Higgs Theory

Ingredients of the theory:

- Compact Lie group G with Lie algebra $\mathfrak{g}$ and Ad-invariant scalar product.
- Finite-dimensional Hilbert space $\mathbf{V}$ with orthogonal action of G.
- Space-time $\mathbf{T}^2$ and "mass" $m \in \mathbf{R}$.

Fields of the theory:

- Gauge field: $\mathfrak{g}$-valued one-forms A on $\mathbf{T}^2$ (actually G-equivariant connections on trivial G-bundle $\mathbf{T}^2 \times G$).
- Higgs field: $\mathbf{V}$-valued field Φ on $\mathbf{T}^2$.

Gauge group $\mathcal{G}$:

- Functions $g \colon \mathbf{T}^2 \to G$ with pointwise multiplication.
- Action on fields: $g \bullet A = \mathrm{Ad}_g A - (dg)g^{-1}$, $g \bullet \Phi = g\Phi$.

Yang–Mills–Higgs Theory

Ingredients of the theory:

- Compact Lie group G with Lie algebra $\mathfrak{g}$ and Ad-invariant scalar product.
- Finite-dimensional Hilbert space $\mathbf{V}$ with orthogonal action of G.
- Space-time $\mathbf{T}^2$ and "mass" $m \in \mathbf{R}$.

Fields of the theory:

- Gauge field: $\mathfrak{g}$-valued one-forms A on $\mathbf{T}^2$ (actually G-equivariant connections on trivial G-bundle $\mathbf{T}^2 \times G$).
- Higgs field: $\mathbf{V}$-valued field Φ on $\mathbf{T}^2$.

Gauge group $\mathcal{G}$:

- Functions $g\colon \mathbf{T}^2 \to G$ with pointwise multiplication.
- Action on fields: $g \bullet A = \mathrm{Ad}_g\, A - (dg)g^{-1}$, $g \bullet \Phi = g\Phi$.

Yang–Mills–Higgs Action

Geometric ingredients: Given A and space $\mathbf{V}$ on which G acts, covariant derivative on $\mathbf{V}$-valued forms:

$$D_A\omega = d\omega + A \wedge \omega \ , \qquad gD_A\omega = D_{g\bullet A}(g\omega) \ .$$

Curvature tensor F^A such that $D_A^2\omega = F^A \wedge \omega$:

$$F^A_{ij} = [D_{A,i}, D_{A,j}] = \partial_i A_j - \partial_j A_i + [A_i, A_j] \ .$$

Gauge invariant action $S(g \bullet A, g\Phi) = S(A, \Phi)$:

$$S(A,\Phi) = \int_{\mathbf{T}^2} \big(|F^A(x)|^2 + |D_A\Phi(x)|^2 + |\Phi(x)|^4 - m^2|\Phi(x)|^2\big)\, dx \ .$$

Yang–Mills–Higgs Action

Geometric ingredients: Given A and space $\mathbf{V}$ on which G acts, covariant derivative on $\mathbf{V}$-valued forms:

$$D_A\omega = d\omega + A \wedge \omega \;, \qquad gD_A\omega = D_{g\bullet A}(g\omega) \;.$$

Curvature tensor F^A such that $D_A^2\omega = F^A \wedge \omega$:

$$F^A_{ij} = [D_{A,i}, D_{A,j}] = \partial_i A_j - \partial_j A_i + [A_i, A_j] \;.$$

Gauge invariant action $S(g \bullet A, g\Phi) = S(A, \Phi)$:

$$S(A,\Phi) = \int_{\mathbf{T}^2} \bigl(|F^A(x)|^2 + |D_A\Phi(x)|^2 + |\Phi(x)|^4 - m^2|\Phi(x)|^2\bigr)\, dx \;.$$

Yang–Mills–Higgs Action

Geometric ingredients: Given A and space $\mathbf{V}$ on which G acts, covariant derivative on $\mathbf{V}$-valued forms:

$$D_A\omega = d\omega + A \wedge \omega \;, \qquad gD_A\omega = D_{g\bullet A}(g\omega) \;.$$

Curvature tensor F^A such that $D_A^2\omega = F^A \wedge \omega$:

$$F_{ij}^A = [D_{A,i}, D_{A,j}] = \partial_i A_j - \partial_j A_i + [A_i, A_j] \;.$$

Gauge invariant action $S(g \bullet A, g\Phi) = S(A, \Phi)$:

$$S(A,\Phi) = \int_{\mathbf{T}^2} \big(|F^A(x)|^2 + |D_A\Phi(x)|^2 + |\Phi(x)|^4 - m^2|\Phi(x)|^2\big)\, dx \;.$$

Yang–Mills–Higgs Action

Geometric ingredients: Given A and space $\mathbf{V}$ on which G acts, covariant derivative on $\mathbf{V}$-valued forms:

$$D_A\omega = d\omega + A \wedge \omega\ , \qquad gD_A\omega = D_{g\bullet A}(g\omega)\ .$$

Curvature tensor F^A such that $D_A^2\omega = F^A \wedge \omega$:

$$F^A_{ij} = [D_{A,i}, D_{A,j}] = \partial_i A_j - \partial_j A_i + [A_i, A_j]\ .$$

Gauge invariant action $S(g \bullet A, g\Phi) = S(A, \Phi)$:

$$S(A,\Phi) = \int_{\mathbf{T}^2} \big(|F^A(x)|^2 + |D_A\Phi(x)|^2 + |\Phi(x)|^4 - m^2|\Phi(x)|^2\big)\, dx\ .$$

Yang–Mills–Higgs Action

Geometric ingredients: Given A and space $\mathbf{V}$ on which G acts, covariant derivative on $\mathbf{V}$-valued forms:

Curvatur

$G = U(1)$ describes electrodynamics: critical points of S solve Maxwell's equations (if metric is Minkowski).

$G = SU(3)$ describes strong force.

Gauge invariant action $S(g \bullet A, g\Phi) = S(A, \Phi)$:

$$S(A, \Phi) = \int_{\mathbf{T}^2} \left(|F^A(x)|^2 + |D_A \Phi(x)|^2 + |\Phi(x)|^4 - m^2 |\Phi(x)|^2 \right) dx \ .$$

Yang–Mills–Higgs Action

Geometric ingredients: Given A and space $\mathbf{V}$ on which G acts, covariant derivative on $\mathbf{V}$-valued forms:

Curvatur

$G = U(1)$ describes electrodynamics: critical points of S solve Maxwell's equations (if metric is Minkowski).

$G = SU(3)$ describes strong force.

Gauge invariant action $S(g \bullet A, g\Phi) = S(A, \Phi)$:

$$S(A, \Phi) = \int_{\mathbf{T}^2} \left(|F^A(x)|^2 + |D_A \Phi(x)|^2 + |\Phi(x)|^4 - m^2 |\Phi(x)|^2 \right) dx \ .$$

Gauge Invariance

Problem: The action functional S is flat in the (infinitely many) directions in which $\mathcal{G}$ acts! Cannot expect to have any $\mathcal{G}$-invariant measure on fields.

Good news: Physical observables $(A, \Phi) \mapsto O(A, \Phi)$ are gauge-invariant, namely $O(g \bullet A, g\Phi) = O(A, \Phi)$ for every $g \in \mathcal{G}$. Two options:

1. Gauge fixing: select one representative in each $\mathcal{G}$-orbit.
2. Work on quotient space.

Geometric obstructions to gauge fixing (and nasty co-area formula). Stochastic quantisation allows to "fix gauge" dynamically.

Gauge Invariance

Problem: The action functional S is flat in the (infinitely many) directions in which $\mathcal{G}$ acts! Cannot expect to have any $\mathcal{G}$-invariant measure on fields.

Good news: Physical observables $(A, \Phi) \mapsto O(A, \Phi)$ are gauge-invariant, namely $O(g \bullet A, g\Phi) = O(A, \Phi)$ for every $g \in \mathcal{G}$. Two options:

1. Gauge fixing: select one representative in each $\mathcal{G}$-orbit.
2. Work on quotient space.

Geometric obstructions to gauge fixing (and nasty co-area formula). Stochastic quantisation allows to "fix gauge" dynamically.

Gauge Invariance

Problem: The action functional S is flat in the (infinitely many) directions in which $\mathcal{G}$ acts! Cannot expect to have any $\mathcal{G}$-invariant measure on fields.

Good news: Physical observables $(A, \Phi) \mapsto O(A, \Phi)$ are gauge-invariant, namely $O(g \bullet A, g\Phi) = O(A, \Phi)$ for every $g \in \mathcal{G}$. Two options:

1. Gauge fixing: select one representative in each $\mathcal{G}$-orbit.
2. Work on quotient space.

Geometric obstructions to gauge fixing (and nasty co-area formula). Stochastic quantisation allows to “fix gauge” dynamically.

Gauge Invariance

Problem: The action functional S is flat in the (infinitely many) directions in which $\mathcal{G}$ acts! Cannot expect to have any $\mathcal{G}$-invariant measure on fields.

Good news: Physical observables $(A, \Phi) \mapsto O(A, \Phi)$ are gauge-invariant, namely $O(g \bullet A, g\Phi) = O(A, \Phi)$ for every $g \in \mathcal{G}$. Two options:

1. Gauge fixing: select one representative in each $\mathcal{G}$-orbit.
2. Work on quotient space.

Geometric obstructions to gauge fixing (and nasty co-area formula). Stochastic quantisation allows to “fix gauge” dynamically.

Gauge Invariance

Problem: The action functional S is flat in the (infinitely many) directions in which $\mathcal{G}$ acts! Cannot expect to have any $\mathcal{G}$-invariant measure on fields.

Good news: Physical observables $(A, \Phi) \mapsto O(A, \Phi)$ are gauge-invariant, namely $O(g \bullet A, g\Phi) = O(A, \Phi)$ for every $g \in \mathcal{G}$. Two options:

1. Gauge fixing: select one representative in each $\mathcal{G}$-orbit.
2. Work on quotient space.

Geometric obstructions to gauge fixing (and nasty co-area formula). Stochastic quantisation allows to "fix gauge" dynamically.

Gauge Invariance

Problem: The action functional S is flat in the (infinitely many) directions in which $\mathcal{G}$ acts! Cannot expect to have any $\mathcal{G}$-invariant measure on fields.

Good news: Physical observables $(A, \Phi) \mapsto O(A, \Phi)$ are gauge-invariant, namely $O(g \bullet A, g\Phi) = O(A, \Phi)$ for every $g \in \mathcal{G}$. Two options:

1. Gauge fixing: select one representative in each $\mathcal{G}$-orbit.
2. Work on quotient space.

Geometric obstructions to gauge fixing (and nasty co-area formula). Stochastic quantisation allows to “fix gauge” dynamically.

Quantisation Equation

For pure Yang–Mills ($\mathbf{V} = \{0\}$), stochastic gradient flow yields

$$\partial_t A = -D_A^* F_A + \xi = -D_A^* D_A A + \frac{1}{2} D_A^* [A, A] + \xi \;,$$

Not parabolic! DeTurck–Donaldson trick: adding $D_A H$ formally preserves dynamic on gauge orbits for any scalar H. Choice $H(A) = -D_A^* A$ yields parabolic system. (Removes $-\partial_{ij}^2 A_j$ and changes l.o.t. in an inessential way)

Basic questions: Interpretation of equation? State space? Gauge equivariance? (Renormalisation formally breaks gauge invariance!)

Quantisation Equation

For pure Yang–Mills ($\mathbf{V} = \{0\}$), stochastic gradient flow yields

$$\partial_t A_i = \Delta A_i - \partial^2_{ij} A_j + \text{l.o.t.} + \xi_i \;.$$

Not parabolic! DeTurck–Donaldson trick: adding $D_A H$ formally preserves dynamic on gauge orbits for any scalar H. Choice $H(A) = -D_A^* A$ yields parabolic system. (Removes $-\partial^2_{ij} A_j$ and changes l.o.t. in an inessential way)

Basic questions: Interpretation of equation? State space? Gauge equivariance? (Renormalisation formally breaks gauge invariance!)

Quantisation Equation

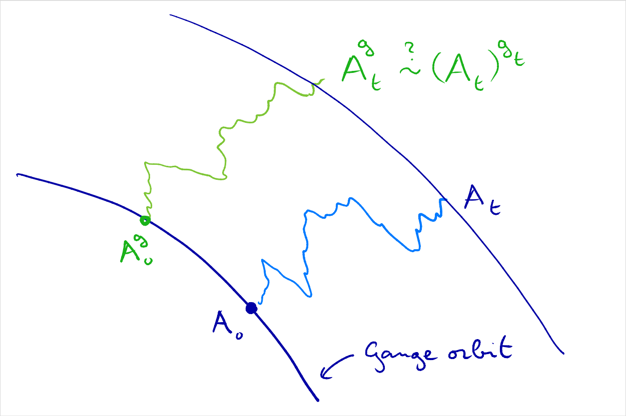
For pure Yang–Mills ($\mathbf{V} = \{0\}$), stochastic gradient flow yields

$$\partial_t A_i = \Delta A_i - \partial^2_{ij} A_j + \text{l.o.t.} + \xi_i \ .$$

Not parabolic! DeTurck–Donaldson trick: adding $D_A H$ formally preserves dynamic on gauge orbits for any scalar H. Choice $H(A) = -D_A^* A$ yields parabolic system. (Removes $-\partial^2_{ij} A_j$ and changes l.o.t. in an inessential way)

Basic questions: Interpretation of equation? State space? Gauge equivariance? (Renormalisation formally breaks gauge invariance!)

Quantisation Equation



For pure Yang–M

Not parabolic! D … preserves
dynamic on gaug … A yields
parabolic system … ssential way)

Basic questions: … equivariance?
(Renormalisation formally breaks gauge invariance.)

Quantisation Equation

For pure Yang–Mills ($\mathbf{V} = \{0\}$), stochastic gradient flow yields

$$\partial_t A_i = \Delta A_i - \partial^2_{ij} A_j + \text{l.o.t.} + \xi_i \;.$$

Not parabolic! DeTurck–Donaldson trick: adding $D_A H$ formally preserves dynamic on gauge orbits for any scalar H. Choice $H(A) = -D^*_A A$ yields parabolic system. (Removes $-\partial^2_{ij} A_j$ and changes l.o.t. in an inessential way)

Basic questions: Interpretation of equation? State space? Gauge equivariance? (Renormalisation formally breaks gauge invariance!)

Local Well-Posedness Result

Theorem (Chandra, Chevyrev, H., Shen '20 & Bringmann, Cao, H., Zhao '26+): Can find Banach space Ω_α of distributional $\mathfrak{g}$-valued 1-forms and space $\mathcal{G}_\alpha$ of Hölder continuous gauge transformations such that:

1. Can renormalise such that $(A_\varepsilon, \Phi_\varepsilon) \to (A, \Phi)$ in probability in $\mathcal{C}(\mathbf{R}_+, \Omega_\alpha \times \mathcal{C}^{-\kappa})$ (modulo possible blow-up).
2. Quotient space $\mathcal{O}_{\alpha,\kappa} = (\Omega_\alpha \times \mathcal{C}^{-\kappa})/\mathcal{G}_\alpha$ is Polish and process is Feller.
3. Wilson loop observables continuous on Ω_α and $\mathcal{G}$-invariant.
4. Unique choice of renormalisation (except for mass of Higgs) such that the quotient process is Markov.

Questions: Global solutions? Uniform bounds? (See Chevyrev–Shen '26 for pure YM and Bringmann–Cao '24 for abelian YMH.)

Local Well-Posedness Result

Theorem (Chandra, Chevyrev, H., Shen '20 & Bringmann, Cao, H., Zhao '26+): Can find Banach space Ω_α of distributional $\mathfrak{g}$-valued 1-forms and space $\mathcal{G}_\alpha$ of Hölder continuous gauge transformations such that:

1. Can renormalise such that $(A_\varepsilon, \Phi_\varepsilon) \to (A, \Phi)$ in probability in $\mathcal{C}(\mathbf{R}_+, \Omega_\alpha \times \mathcal{C}^{-\kappa})$ (modulo possible blow-up).
2. Quotient space $\mathcal{O}_{\alpha,\kappa} = (\Omega_\alpha \times \mathcal{C}^{-\kappa})/\mathcal{G}_\alpha$ is Polish and process is Feller.
3. Wilson loop observables continuous on Ω_α and $\mathcal{G}$-invariant.
4. Unique choice of renormalisation (except for mass of Higgs) such that the quotient process is Markov.

Questions: Global solutions? Uniform bounds? (See Chevyrev–Shen '26 for pure YM and Bringmann–Cao '24 for abelian YMH.)

Local Well-Posedness Result

Theorem (Chandra, Chevyrev, H., Shen ’20 & Bringmann, Cao, H., Zhao ’26+): Can find Banach space Ω_α of distributional $\mathfrak{g}$-valued 1-forms and space $\mathcal{G}_\alpha$ of Hölder continuous gauge transformations such that:

1. Can renormalise such that $(A_\varepsilon, \Phi_\varepsilon) \to (A, \Phi)$ in probability in $\mathcal{C}(\mathbf{R}_+, \Omega_\alpha \times \mathcal{C}^{-\kappa})$ (modulo possible blow-up).
2. Quotient space $\mathcal{O}_{\alpha,\kappa} = (\Omega_\alpha \times \mathcal{C}^{-\kappa})/\mathcal{G}_\alpha$ is Polish and process is Feller.
3. Wilson loop observables continuous on Ω_α and $\mathcal{G}$-invariant.
4. Unique choice of renormalisation (except for mass of Higgs) such that the quotient process is Markov.

Questions: Global solutions? Uniform bounds? (See Chevyrev–Shen ’26 for pure YM and Bringmann–Cao ’24 for abelian YMH.)

Local Well-Posedness Result

Theorem (Chandra, Chevyrev, H., Shen '20 & Bringmann, Cao, H., Zhao '26+): Can find Banach space Ω_α of distributional $\mathfrak{g}$-valued 1-forms and space $\mathcal{G}_\alpha$ of Hölder continuous gauge transformations such that:

1. Can renormalise such that $(A_\varepsilon, \Phi_\varepsilon) \to (A, \Phi)$ in probability in $\mathcal{C}(\mathbf{R}_+, \Omega_\alpha \times \mathcal{C}^{-\kappa})$ (modulo possible blow-up).
2. Quotient space $\mathcal{O}_{\alpha,\kappa} = (\Omega_\alpha \times \mathcal{C}^{-\kappa})/\mathcal{G}_\alpha$ is Polish and process is Feller.
3. Wilson loop observables continuous on Ω_α and $\mathcal{G}$-invariant.
4. Unique choice of renormalisation (except for mass of Higgs) such that the quotient process is Markov.

Questions: Global solutions? Uniform bounds? (See Chevyrev–Shen '26 for pure YM and Bringmann–Cao '24 for abelian YMH.)

Local Well-Posedness Result

Theorem (Chandra, Chevyrev, H., Shen '20 & Bringmann, Cao, H., Zhao '26+): Can find Banach space Ω_α of distributional $\mathfrak{g}$-valued 1-forms and space $\mathcal{G}_\alpha$ of Hölder continuous gauge transformations such that:

1. Can renormalise such that $(A_\varepsilon, \Phi_\varepsilon) \to (A, \Phi)$ in probability in $\mathcal{C}(\mathbf{R}_+, \Omega_\alpha \times \mathcal{C}^{-\kappa})$ (modulo possible blow-up).
2. Quotient space $\mathcal{O}_{\alpha,\kappa} = (\Omega_\alpha \times \mathcal{C}^{-\kappa})/\mathcal{G}_\alpha$ is Polish and process is Feller.
3. Wilson loop observables continuous on Ω_α and $\mathcal{G}$-invariant.
4. Unique choice of renormalisation (except for mass of Higgs) such that the quotient process is Markov.

Questions: Global solutions? Uniform bounds? (See Chevyrev–Shen '26 for pure YM and Bringmann–Cao '24 for abelian YMH.)

Local Well-Posedness Result

Theorem (Chandra, Chevyrev, H., Shen '20 & Bringmann, Cao, H., Zhao '26+): Can find Banach space Ω_α of distributional $\mathfrak{g}$-valued 1-forms and space $\mathcal{G}_\alpha$ of Hölder continuous gauge transformations such that:

1. Can renormalise such that $(A_\varepsilon, \Phi_\varepsilon) \to (A, \Phi)$ in probability in $\mathcal{C}(\mathbf{R}_+, \Omega_\alpha \times \mathcal{C}^{-\kappa})$ (modulo possible blow-up).
2. Quotient space $\mathcal{O}_{\alpha,\kappa} = (\Omega_\alpha \times \mathcal{C}^{-\kappa})/\mathcal{G}_\alpha$ is Polish and process is Feller.
3. Wilson loop observables continuous on Ω_α and $\mathcal{G}$-invariant.
4. Unique choice of renormalisation (except for mass of Higgs) such that the quotient process is Markov.

Questions: Global solutions? Uniform bounds? (See Chevyrev–Shen '26 for pure YM and Bringmann–Cao '24 for abelian YMH.)

Main result

Theorem (Bringmann, Cao, H., Zhao '26+): Solutions are global and the process admits a unique invariant probability measure on $\mathcal{O}_{\alpha,\kappa}$.

Some of the problems one faces:

1. No global theory for gradient flow with DeTurck term even without noise! (Could potentially explode along gauge orbits.)
2. No local theory without DeTurck term in noisy case…
3. Yang–Mills connections of high curvature can get gradient flow stuck.
4. Energy $S(A, \Phi)$ requires $1 + \varepsilon$ more derivatives to be defined!

Main result

Theorem (Bringmann, Cao, H., Zhao ’26+): Solutions are global and the process admits a unique invariant probability measure on $\mathcal{O}_{\alpha,\kappa}$.

Some of the problems one faces:

1. No global theory for gradient flow with DeTurck term even without noise! (Could potentially explode along gauge orbits.)
2. No local theory without DeTurck term in noisy case...
3. Yang–Mills connections of high curvature can get gradient flow stuck.
4. Energy $S(A,\Phi)$ requires $1+\varepsilon$ more derivatives to be defined!

Main result

Theorem (Bringmann, Cao, H., Zhao '26+): Solutions are global and the process admits a unique invariant probability measure on $\mathcal{O}_{\alpha,\kappa}$.

Some of the problems one faces:

1. No global theory for gradient flow with DeTurck term even without noise! (Could potentially explode along gauge orbits.)
2. No local theory without DeTurck term in noisy case...
3. Yang–Mills connections of high curvature can get gradient flow stuck.
4. Energy $S(A,\Phi)$ requires $1+\varepsilon$ more derivatives to be defined!

Main result

Theorem (Bringmann, Cao, H., Zhao '26+): Solutions are global and the process admits a unique invariant probability measure on $\mathcal{O}_{\alpha,\kappa}$.

Some of the problems one faces:

1. No global theory for gradient flow with DeTurck term even without noise! (Could potentially explode along gauge orbits.)
2. No local theory without DeTurck term in noisy case...
3. Yang–Mills connections of high curvature can get gradient flow stuck.
4. Energy $S(A, \Phi)$ requires $1 + \varepsilon$ more derivatives to be defined!

Main result

Theorem (Bringmann, Cao, H., Zhao '26+): Solutions are global and the process admits a unique invariant probability measure on $\mathcal{O}_{\alpha,\kappa}$.

Some of the problems one faces:

1. No global theory for gradient flow with DeTurck term even without noise! (Could potentially explode along gauge orbits.)
2. No local theory without DeTurck term in noisy case...
3. Yang–Mills connections of high curvature can get gradient flow stuck.
4. Energy $S(A, \Phi)$ requires $1 + \varepsilon$ more derivatives to be defined!

Ideas

Gauge: Use reference connection B solving YM gradient flow and apply DeTurck only to $A - B$ (effectively DeTurck for high frequencies).

Local Picard iteration: Work as much as possible in a gauge covariant way. Use covariant smoothing of noise and covariant paracontrolled calculus. Yields local solutions on 'long' intervals and covariant renormalisation. (But breaks Markov property! Restored only for 'correct' renormalisation.)

Control: Use gauge-invariant "norms" based on behaviour under covariant heat flow with connection B. Behave well under gradient flow without DeTurck.

Breaking free: Exploit noise to show that one gets unstuck from near YM connections.

Ideas

Gauge: Use reference connection B solving YM gradient flow and apply DeTurck only to $A - B$ (effectively DeTurck for high frequencies).

Local Picard iteration: Work as much as possible in a gauge covariant way. Use covariant smoothing of noise and covariant paracontrolled calculus. Yields local solutions on ‘long’ intervals and covariant renormalisation. (But breaks Markov property! Restored only for ‘correct’ renormalisation.)

Control: Use gauge-invariant “norms” based on behaviour under covariant heat flow with connection B. Behave well under gradient flow without DeTurck.

Breaking free: Exploit noise to show that one gets unstuck from near YM connections.

Ideas

Gauge: Use reference connection B solving YM gradient flow and apply DeTurck only to $A - B$ (effectively DeTurck for high frequencies).

Local Picard iteration: Work as much as possible in a gauge covariant way. Use covariant smoothing of noise and covariant paracontrolled calculus. Yields local solutions on 'long' intervals and covariant renormalisation. (But breaks Markov property! Restored only for 'correct' renormalisation.)

Control: Use gauge-invariant "norms" based on behaviour under covariant heat flow with connection B. Behave well under gradient flow without DeTurck.

Breaking free: Exploit noise to show that one gets unstuck from near YM connections.

Ideas

Gauge: Use reference connection B solving YM gradient flow and apply DeTurck only to $A - B$ (effectively DeTurck for high frequencies).

Local Picard iteration: Work as much as possible in a gauge covariant way. Use covariant smoothing of noise and covariant paracontrolled calculus. Yields local solutions on 'long' intervals and covariant renormalisation. (But breaks Markov property! Restored only for 'correct' renormalisation.)

Control: Use gauge-invariant "norms" based on behaviour under covariant heat flow with connection B. Behave well under gradient flow without DeTurck.

Breaking free: Exploit noise to show that one gets unstuck from near YM connections.

Some open questions

- Global solutions / uniform bounds on $\mathbf{R}^2$?
- Global solutions in 3D for YM or YMH?
- Mass gap for $SU(N)$ in 3D, at least for large N?
- "Good" spaces of observables in 3D and 4D?
- Renormalised Wilson loop observables in 3D?

Thanks for your attention!

See Wenhao's poster outside of the lecture room.

Some open questions

- Global solutions / uniform bounds on $\mathbf{R}^2$?
- Global solutions in 3D for YM or YMH?
- Mass gap for $SU(N)$ in 3D, at least for large N?
- "Good" spaces of observables in 3D and 4D?
- Renormalised Wilson loop observables in 3D?

Thanks for your attention!

See Wenhao's poster outside of the lecture room.